Chapter 5 — Materials

Phil Putman
July 2006

Electromagnetic launchers often operate near material limits. This chapter contains some information about the current-carrying materials that can be used in a launcher, both conventional and superconducting, and the relative merits of each.

Copper and other conventional conductors

A potential advantage of superconductors over conventional copper conductors is the capability to operate at high current densities. (Others advantages are high efficiency and the possible exploitation of persistent currents.) Superconductors, however, do not necessarily outperform normal conductors in all pulsed power applications. The following quote from [Mongeau], which predates high temperature superconductivity, outlines the situation:

> With the widespread use of superconducting systems it is often questioned why high performance accelerators are not all superconducting thus eliminating all the problems of resistive heating in the first place. Aside from the obvious complexity of the required cryogenic systems and the problems of induced heating… there is the often overlooked observation that a normal conductor will easily outperform a superconducting one if the acceleration durations are sufficiently short. Niobium tin (Nb_3Sn) can sustain current densities in excess of 200 kA/cm^2, but necessary stabilization material will reduce this effectively to 25 kA/cm^2. Copper, on the other hand, has no intrinsic current density limit and can easily sustain this current density as long as its thermal inertia will allow. For a current integral of 4×10^8 A^{2}s/cm^4 (corresponding to a temperature rise from room temp. of 300°C) copper will carry 25 kA/cm^2 for about half a second. In other words if the times of inter-

est are less than half a second it is usually desirable to use a normal conductor system (the break point from aluminum is 0.3 seconds).

In most launcher topologies, the armature must sustain the current for the duration of the launch, while the stationary part operates in sections for shorter times. When comparing High Temperature Superconductor (HTS) materials to copper, this duration must be taken into account.

For conventional conductors, he energy deposited per unit time is (see [Mongeau])

$$\frac{dQ}{dt} = \rho J^2 \qquad (1)$$

where Q is the energy density, ρ is the resistivity, and J is the current density.

For adiabatic heating eq. 1 can be expressed as

$$C_V \frac{dT}{dt} = \rho J^2 . \qquad (2)$$

Since C_V and ρ are usually functions of temperature, eq. 2 can be expressed as

$$J^2 = \frac{C_V[t]}{\rho[t]} \frac{dT}{dt} . \qquad (3)$$

Integrating produces

$$\int_0^t J^2 dt = \int_{T_i}^{T_f} \frac{C_V}{\rho} dT , \qquad (4)$$

which relates the current history of a sample to its temperature. This current integral has been calculated explicitly for a variety of materials.

If the resistivity as a function of temperature is modeled as

$$\rho = \rho_0[T_i](1 + \alpha(T_f - T_i)) \qquad (5)$$

and the specific heat as a constant, eq. 3 reduces to

$$\int_0^t J^2 dt = \int_{T_i}^{T_f} \frac{C_V}{\rho_0} \frac{1}{1+\alpha(T_f - T_i)} dT \tag{6}$$

or

$$J^2 t = \int_0^{\Delta t} \frac{C_V}{\rho_0} \frac{1}{1+\alpha T} dT = \frac{C_V}{\rho_0 \alpha} \ln[1+\alpha \Delta T] , \tag{7}$$

where $\Delta T = T_f - T_i$. This "action" approach is recommended in [Anderson], and is the standard criterion used in specifying fuses (see [Wright]).

For most purposes the melting point of the conductor represents a rigid limit. Practically speaking, the conductor temperature must be maintained to some point substantially below this for strength reasons.

Mongeau cites the J^2t for copper from room temperature to the melting point (with no melting) to be 8.9 X10^8 A^2s/cm^4. Using ρ at 293K = 1.7 X10^{-6} Ωcm, α = 0.004 K^{-1}, C_V = 3.4265 J/cm^3K, eq. 7 produces J^2t[300 K] = 9.96 X10^8 A^2s/cm^4, J^2t[77 K] = 7.07 X10^9 A^2s/cm^4. This implies that the maximum J for a 1 s launch is 31 kA/cm^2 at 300 K, or 84 kA/cm^2 with the Cu precooled to 77K.

J_C of readily available HTS monoliths is 30 kA/cm^2, and of ion-irradiated monoliths, 270 kA/cm^2. The present world record holder has set a goal of 1 MA/cm^2 monoliths. J_C for HTS wires, including their substrate, is about 50 kA/cm^2. The conclusion, therefore, is that both currently available monoliths and coated conductors are superior for use in armatures, and, if progress continues, monoliths will be superior for use in stationary parts as well based solely on their current density limit.

Available superconductors

Researchers have discovered many materials that are superconducting at the boiling temperature of liquid nitrogen. Table 5-I lists some of the commonly studied compounds, and the temperature below which they are superconducting. The fact that some of these materials have zero resistance at the relatively easily produced temperature of 77 K is a first step toward their use in commercial products. However, other properties are also important. Most applications require the superconductor to operate in a magnetic field, possibly self-generated. As discussed below, the presence of a magnetic field can cause resistance. The field above which this occurs is called the irreversibility field, and is also listed in Table 5-I. This field is, to a certain extent, an extrinsic property, which depends on the presence of imperfections in the material. However, there is an upper limit to the improvement that can be made by tailoring the imperfections. All of the high-temperature superconductors are layered materials. Materials with large distance between layers, such as BSCCO, have lower maximum irreversibility fields.

Table 5-I

Material	T_c (K)	B_{irr} (T)
Nb-Ti	9.5	13 (@4.2K)
Nb_3Sn	18.4	22
V_3Ga	15	23
$Nb_3(Al_{75}Ge_{75})$	21	40
Nb_3Ge	23.2	36
MgB_2	39	15
$YBa_2Cu_3O_{7-\delta}$	92	5 (@77K)
$Bi_2Sr_2CaCu_2O_x$	90	<0.1
$Bi_2Sr_2Ca_2Cu_3O_y$	105	0.3
$Tl_2Ca_2Ba_2Cu_3O_z$	120	4

The layered structure of these materials also has a strong effect on their mechanical properties. Because of their small coherence length, the materials must be used in approximately single-crystal form, so their anisotropy is not averaged as it is in fine-grained materials. The ratio of some properties in the c direction compared to the direction parallel to the ab planes is called the anisotropy parameter γ, given by

$$\gamma = \left(\frac{m_c}{m_{ab}}\right)^{1/2} = \frac{\lambda_c}{\lambda_{ab}} = \frac{\xi_{ab}}{\xi_c} = \frac{H_{c2\|ab}}{H_{c2\|c}} = \frac{H_{c1\|c}}{H_{c1\|ab}},$$

where m_c and m_{ab} are the effective masses in the *c* and *ab* directions, λ is the penetration length, ξ is the coherence length, H_{C2} is the upper critical magnetic field, and H_C is the thermodynamic critical field. For YBCO, γ is approximately 7, while for BSCCO, more than 150. Reviews of high-temperature superconductivity can be found in many textbooks, for example [Tinkham] and [Sheahen]. The remainder of this section will focus on the electrical and magnetic properties that are important for high-current applications.

Transport current in superconductors

Superconductors acquired their name due to their ability to carry current with no resistance under certain conditions. This state is due to electron pairing, which has the effect of eliminating mechanisms that would allow the current to decrease.

Electron pairs, referred to as Cooper pairs after one of the first scientists to propose their existence, occur because of an attractive force between the electrons. In conventional superconductors, the attractive force is caused by electron-lattice interactions. In high-temperature superconductors, there is not yet agreement on the source of pairing.

In conventional superconductors, electrons near the Fermi surface with the same magnitude of momentum, but opposite direction and spin, are paired. This causes an energy gap, Δ, between the top of the ground state and the lowest available excited states. When electrons carry a current, the entire Fermi surface moves by an amount δk. The energy gap moves with the Fermi surface. In a normal conductor, individual electrons can be scattered from one side of the Fermi surface to another by interaction with a phonon or an impurity. Changing the momentum in a superconductor requires changing the momentum of two electrons. Energy Δ must be supplied from the phonon or impurity, which in most cases is more than is available. Therefore, although the energy of the system would be reduced if somehow the entire Fermi surface could move back to its original location centered on zero momentum, there is no mechanism available for this to occur.

At a large enough current, it is no longer energetically favorable for the electrons to remain paired. This occurs at a current density of

$$J_c = \frac{e n_S \Delta}{p_F},$$

where e is the charge of an electron, n_S is the superconducting electron density, Δ is the pairing energy, and p_F is the Fermi momentum. The depairing current is an upper limit on current flow without resistance. Before this current is reached, however, flux lines begin to move and cause a resistance. [Kunchur]

In type-II superconductors, the group to which all HTS materials belong, the Ginzburg-Landau parameter, given by

$$\kappa = \frac{\lambda}{\xi},$$

which is the ratio of penetration depth λ to coherence length ξ, is larger than 0.71. This implies that the surface energy for regions of nonzero magnetic field inside the superconductor is negative. It is therefore energetically favorable for as many individual regions of nonzero field as possible to form. In superconductors, flux is quantized, with the unit of flux given by

$$\Phi_0 = \frac{h}{2e},$$

Where h is Planck's constant and e is the charge of an electron. The field will therefore penetrate as individual flux lines of one flux quantum. The Lorentz force,

$$\mathbf{F} = \mathbf{J} \times \mathbf{B},$$

where **J** is the current density and **B** is the magnetic field, acts on these fluxoids. This force causes the fluxoids to move transverse to the current. For velocity **v**, the induced electric field is given by

$$\mathbf{E} = \mathbf{B} \times \mathbf{v},$$

which is parallel to **J**. The flux flow resistance can be illustrated by the examination of a model problem, shown in Fig. 6-1. The Lorentz force causes the fluxoids to move outward through the thin wall of the cylinder, reducing the flux inside. The induced EMF is given by

$$V = 2\pi r E = -\frac{d\Phi}{dt} = 2\pi r B v,$$

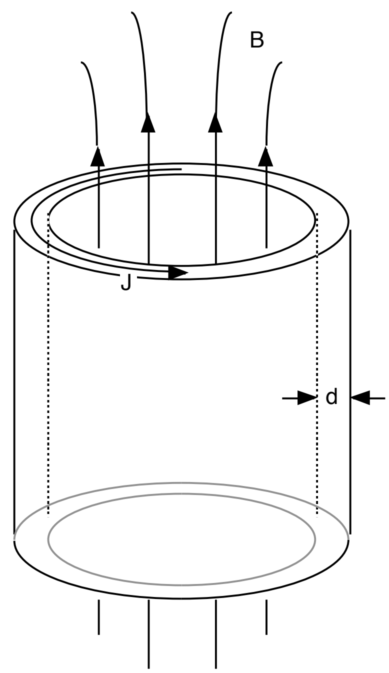

Figure 6-1 Trapped field in cylinder

where E is the electric field and Φ is the magnetic flux through the cylinder. This implies that the induced field is equal to Bv.

Bardeen and Stephen found the flux flow resistivity by first deriving an expression for the flux flow viscous drag coefficient. The power dissipation due to flux motion is

$$P = \eta v^2 \qquad (8),$$

where η is the flux flow viscosity. If the electric field E can be found, the expression for dissipation can be set equal to

$$P = E^2 / \rho,$$

where ρ is the flux flow resistivity, and solved for η in terms of given variables. Some simplifying assumptions are necessary to make the problem tractable. Fluxoids have a normal core, with a gradual transition to the superconducting state that would exist in the absence of flux penetration. This can be approximated by a normal core of radius ξ, with a sharp transition from the normal state to the superconducting state. Ohm's law will apply inside the core, and the London equations outside. For a discussion of the London equations, see [Rose-Innes] or [Tinkham]. The local field **e** outside the core can be found using the first London equation (see Fig. 6-2),

$$\mathbf{e} = \frac{\partial}{\partial t}(\Lambda \mathbf{J}_s) = \frac{\partial}{\partial t}\left(\frac{m^* \mathbf{v}_s}{e^*}\right) = -\mathbf{v}\cdot\nabla\left(\frac{m^* \mathbf{v}_s}{e^*}\right) = -\mathbf{v}\cdot\nabla\left(\frac{\hbar}{2e}\frac{\hat{\theta}}{r}\right).$$

For **v** along the $\hat{x}$ direction,

$$\mathbf{e} = -\frac{v\Phi_0}{2\pi}\frac{\partial}{\partial x}\left(\frac{\hat{\theta}}{r}\right) = \frac{v\Phi_0}{2\pi r^2}(Cos[\theta]\hat{\theta} - Sin[\theta]\hat{r}) \qquad (9),$$

The field inside the core can be found by requiring continuity at $r = a$, and gives

$$\mathbf{e}_{\text{core}} = \frac{v\Phi_0}{2\pi a^2}\hat{y}.$$

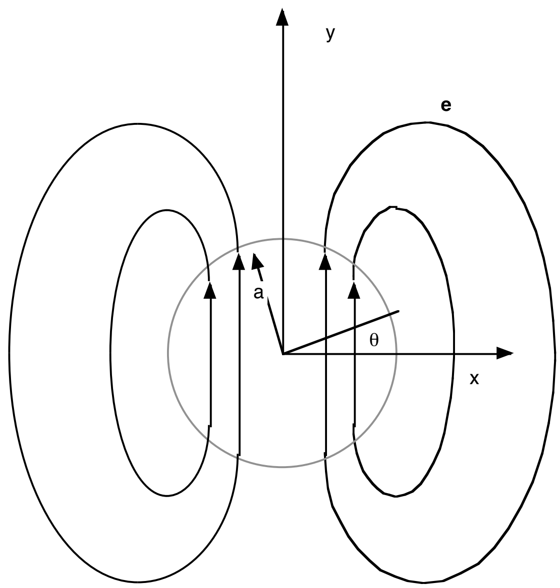

Figure 6-2 Local electric field near a moving fluxoid

The dissipation per unit volume is

$$P = \pi a^2 \sigma_n e^2{}_{\text{core}} = \frac{v^2 \Phi^2{}_0}{4\pi a^2 \rho_n} \hat{y}$$
.

Integrating eq. 9 outside the core gives an equal amount of dissipation in the transition region outside the core (if the conductivity near the core is approximately equal to the normal resistivity). Now, setting these losses equal to eq.8, the flux flow viscosity is given by

$$\eta = \frac{\Phi^2{}_0}{2\pi a^2 \rho_n} \approx \frac{\Phi_0 \mu_0 H_{c2}}{\rho_n} \qquad (10).$$

Φ_0 is the quantum of magnetic flux, μ_0 is the permeability of vacuum, H_{C2} is the upper critical field, and ρ_n is the normal state resistivity. The force per length on a fluxoid is $-\eta v$. The driving force is $J\Phi$, so, in the steady state,

$$\eta v = J\Phi .$$

This can be substituted into the expression the Lorentz force equation to produce

$$\rho_f = \frac{E}{J} = B\frac{\Phi_0}{\eta} .$$

Finally, combining this equation with eq. 10 gives the flux flow resistivity,

$$\rho_f = \rho_n \frac{2\pi a^2 B}{\Phi_0} \approx \frac{B}{\mu_0 H_{c2}} .$$

Dynamic effects might be seen in flux flow under some circumstances. For example, at very short time scales, fluxoids could exhibit inertial effects. However, performing experiments at such short time scales results in the destruction of the sample, and so the phenomenon is not well understood.

Flux flow is prevented if the Lorentz force is less than a pinning force caused by nonsuperconducting regions. The effectiveness of a pinning center depends on its shape, with line-type centers aligned with the flux being the most effective. Long pinning centers can be produced by bombardment with energetic ions or fission of radioactive inclusions. Spherical inclusions of Y_2BaCuO_6 are also effective in pinning lines of flux. For more on flux pinning, see [Tinkham].

Methods of modeling superconductors

Superconducting materials require a constitutive model between current J, magnetic field B, and electric field E. This model generally includes the parameter J_c, critical current density.

Possibly the best known model of a monolithic superconductor is the Bean model [Bean]. The key features of this model are a constant critical current density, J_c, throughout the sample, and that the current everywhere in the sample has a value of +/- J_c or 0. This model is useful because of its simplicity, and gives approximately accurate results in some situations such as application and removal of uniform magnetic fields. However, in some situations that arise in launchers, the second assumption is grossly inaccurate. For example, if a monolithic magnet begins in its critical state and the applied field is gradually decreased, the current in the outer portion of the magnet also decreases gradually instead of reversing instantly to $-J_c$.

The other simplification of the Bean model, i.e. J_c is independent of B, is abandoned in the more accurate Kim model [Kim]. In the Kim model, J_c is given by

$$J_c = \frac{\alpha}{B_0 + B},$$

where α is related to current carrying capacity, B_0 is approximately $\mu_0 H_c$, and B is the applied field. The disadvantage of this model is that iteration is required, since B also depends on J. It has been found that in some cases a field-independent value of J_c which is one-half the zero field value can be used with good accuracy [Jiang].

Of course, J can also exceed J_c. The constitutive relation is broken into several regimes in. [Kunchur]. Depending on J, ρ (=E/J) is given by a different equation. While J is less than J_c, ρ is dominated by thermally activated flux flow (sometimes called flux creep). The relation in this region is given by

$$E = J\left(\frac{2BL\Omega U_0}{cJ_{c0}k_B T}\right)e^{-U_0 / k_B T},$$

where L is the distance between pinning sites, Ω is the frequency at which vortices attempt to leave their pinning site, and U_0 is the depth of the potential barrier. This regime is not important in an EM launcher, as the time constant for a magnet in this regime is much larger than launch time.

The next regime, in which $J > J_c$, is the flux flow regime. In this regime, ρ is given by

$$\rho = \rho_n \frac{B}{H_{c2}[T]}$$

where ρ_n is the normal state resistivity. This is the Bardeen-Stephen equation, and it has been found to agree with experiment in YBCO. [Tinkham]

In the next regime, the normal state, the kinetic energy of the electrons exceeds the condensation energy. The value of the condensation energy can be calculated from a measurement of the critical field:

$$J_d = \frac{cH_c}{4\pi\lambda}.$$

This result is due to the London approach, with λ the London penetration depth. A more accurate calculation can be performed using Ginsburg-Landau theory, which results in a depinning current of 0.54 times the London value. At high enough current densities, the sample will crack, melt, and/or vaporize.

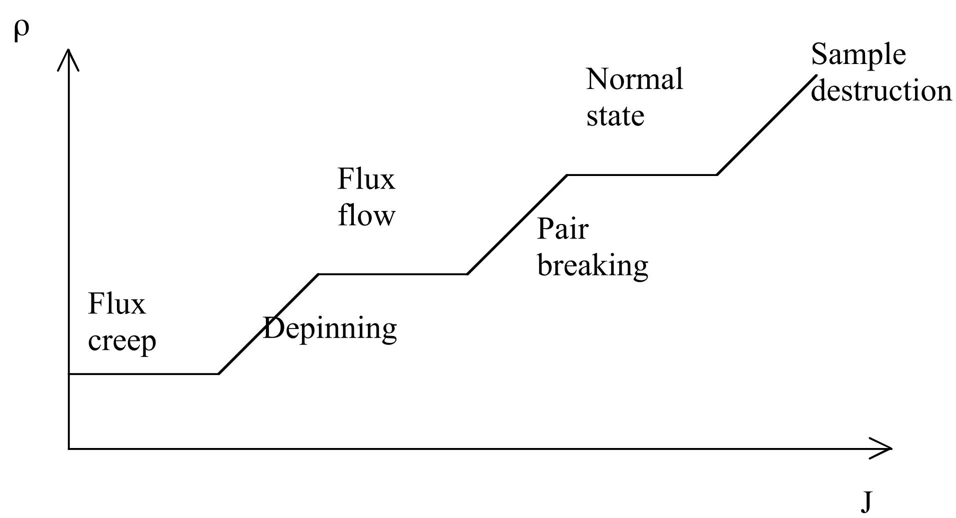

Figure 6-3

References

P. Mongeau, *Coaxial Air Core Electromagnetic Accelerators*, M.I.T. Doctoral Dissertation, 1982, p. 97.

G.W. Anderson and F.W. Neilson, "Use of the 'action integral' in exploding wire studies," *Exploding Wires*, W.G. Chace and H.K. Moore, eds., New York: Plenum Press, 1959, pp. 97–103.

A. Wright and P.G. Newbury, *Electric Fuses*, The Institution of Electrical Engineers, London, 1995, p. 114.

E.C. Cnare, "Magnetic flux compression by magnetically imploded metal foils," *Journal of Applied Physics*, vol. 37, pp. 3812–3816 (1966).

M. Tinkham, *Introduction to Superconductivity*. New York: McGraw-Hill, 1996.

T.P. Sheahen, *Introduction to High-Temperature Superconductivity*. New York: Plenum Press, 1994.

M.N. Kunchur, "Novel transport behavior found in the dissipative regime of superconductors," *Modern Physics Letters B*, vol. 9 (7), pp. 399-426, 1995.

A.C. Rose-Innes and E.H. Rhoderick, *Introduction to Superconductivity*. Tarrytown, N.Y.: Elsevier Science Inc., 1978.

X.H. Jiang and A.M. Campbell, "Numerical Calculation of Magnetic Fields in Melt Processed YBCO Magnets," *IEEE Trans. Appl. Superconductivity*, vol. 7, no. 2, pp. 1213-1215, 1997.

C.P. Bean, "Magnetization of High-Field Superconductors," *Reviews of Modern Physics*, vol. 36, pp. 31-39, 1964.

Y.B. Kim, C.F. Hempstead, and A.R. Strnad, "Magnetization and Critical Supercurrents," *Phys. Rev.*, vol. 129, no.2, pp. 528-535, 1963.